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ABSTRACT

Investigated was the development of a diagnostic and individualized remedial program for prospective elementary teachers in the area of mathematical skills. Subjects were 120 students in the pre-methods course on number systems during the Winter and Spring 1971 terms at the Ogontz Campus, Pennsylvania State University. All subjects were given a series of tests prior to beginning the course, and were retested with the same tests at the end of the course. Students scoring below criterion on the pre-test diagnostic test were randomly assigned to an experimental group for remedial treatment or to a control group. Post-testing revealed an increase in diagnostic scores, some improvement in self-evaluation, and an improvement in attitude scores. However, no significant differences in test score gains were found between the experimental and the control groups. Because students in the experimental group attended the remedial clinic in very irregular patterns, the investigator concludes that test gains were not related to the experimental treatment. It was found possible to diagnose arithmetic deficiencies and to prescribe appropriate remedial treatment, although carrying through the remediation program posed several problems. Appended are facsimiles of test materials utilized. (JG)

Final Report
Project No. 0B058
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The Diagnosis and Remediation of Mathematical Skills for Prospective Elementary School Teachers

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U. S. Department of Health, Education and Welfare
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(Regional Research Programs)

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U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
National Center for Educational Research and Development

PREFACE

Significant cooperation and assistance were given to the investigators by Dr. Marilyn N. Suydam, Acting Director of the Center for Cooperative Research with Schools (CReWS) at the University Park Campus, and by Mrs. Patricia Overdeer, Associate Professor of Mathematics at the Ogontz Campus, both The Pennsylvania State University. Their contributions to the project are gratefully acknowledged.

ABSTRACT

The problem to be investigated was the development of a diagnostic and individualized remedial program for prospective elementary teachers in the area of mathematics skills. Subjects in the investigation were 120 students enrolled in the pre-methods course on number systems (Mathematics 200) during the Winter and Spring 1971 terms at the Ogontz Campus, The Pennsylvania State University. All subjects were given a series of tests prior to beginning the course, and were retested with the same tests at the end of the course. Students scoring below criterion on the pre-test diagnostic test were randomly assigned to an experimental group for remedial treatment or to a control group. Post-testing revealed an increase in diagnostic scores unrelated to the experimental treatment, some improvement in self-evaluation, and an improvement in attitude scores.

In conclusion, the investigators have found that it is possible to diagnose arithmetic deficiencies and to prescribe appropriate remedial treatment, although carrying through the remediation program poses several problems. A follow-up study of the durability of changes which have occurred is desirable as the subjects progress to the methods course in teaching arithmetic and then to student teaching.

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Introduction

Researchers in elementary school mathematics education and others interested in this area have found that the majority of elementary school teachers still lack basic mathematical skills and concepts necessary for effective teaching of the subject. Furthermore, in interviews, elementary education majors at the Ogontz Campus have frequently expressed their feelings of incompetence in and aversion to mathematics. This situation was very distrubing to the Education and Mathematics faculties at the Campus.

Teachers of mathematics courses in which our students are required to enroll have often found them incapable of dealing with, or completely ignorant of, fundamental arithmetic techniques. The investigators felt that if permitted to continue their educational programs with no improvement in competence, these students will become teachers who are ill-prepared to teach arithmetic to their pupils. The major purposes of the project thus became (1) to develop a means to reduce the frequency with which cycles of mathematical deficiency and negative attitudes beget further deficiences and poor attitudes, and (2) to enhance the actual and self-perceived competence of prospective elementary school teachers in mathematics.

At The Pennsylvania State University (including branch campuses such as Ogontz), a three-credit course on number systems (hereafter referred to as Math 200) is required of elementary education majors for graduation and certification, and is scheduled prior to the methods course in teaching mathematics. This course is designed to explain concepts underlying arithmetic operations, not to teach the operations themselves. However, the backgrounds of students vary considerably, from those well-prepared to comprehend the concepts to those ill-prepared to apply them. As more complex concepts are introduced earlier in the elementary school mathematics curriculum, it becomes imperative that the teacher be more secure in his knowledge of the basic operations, so that he can communicate both the skills and the concepts to his pupils. It appeared vital, therefore, to remedy or at least moderate existing mathematical deficiences.

Researchers report varied findings on the attitudes of pre-service teachers toward mathematics. Smith (1964) reported that they were in the majority favorable. Dutton (1962) found that there were slightly more favorable attitudes after mathematics courses. Kane (1968) and Smith (1964) found that attitudes were unfavorable.

Unfavorable attitudes were related to lack of understanding, insecurity and fear of making mistakes, and difficulty, while favorable attitudes were related to enjoyment, importance, challenge, and good teachers (Dutton, 1951, 1954, 1962)

Improved competence will bring a dimension of confidence to the teacher's classroom approach and create a more positive attitude during mathematics lessons. Hopefully, increases in self-confidence and positive attitudes on the part of the teacher will ultimately be reflected in the attitudes and achievement of his/her puplis.

With these ideas in mind, the investigators began a search for measurement instruments which would be diagnostic and would differentiate between levels of comptence among the elementary education majors. A number of problems quickly became evident in the survey of published texts: (1) they were inappropriate for use with college students, (2) they were not useful for diagnostic purposes, and/or (3) they were handscored. The first task of the investigators, therefore, was to develop a diagnostic test which could be machine-scored but still reveal specific weaknesses. Until the diagnostic test was developed, the Wide-Range Achievement Test (WRA) was used to estimate the extent of mathematics deficiencies.

Students were informed of their weak areas, and of available remedial material. In addition, a weekly one-hour mathematics clinic was held during each of two ten-week terms (Winter and Spring 1971); and experimental group of weaker students was urged to attend.

Research evidence on the background on in-service teachers was helpful in planning the remedial program. Dutton and Hammond (1966), in particular, found that identification of weaknesses which teachers have in understanding mathematics, and then teaching adapted to individual needs to overcome these weaknesses, were most effective. Programmed instructional materials also appeared to be effective diagnostic and remedial tools.

Goals of the study were to try to answer the following questions: A. Does an individualized remedial program promote achievement in (1) basic mathematical skills? (2) Mathematics 200? B. What is the attitude toward mathematics of students (1) upon entering the elementary education sequence? (2) before Mathematics 200? (3) during the remedial program? (4) after the remedial program? (5) after Mathematics 200? C. What are the reactions

of students to (1) the diagnostic test? (2) the programmed remedial materials? (3) remediation at the college level?

Not all of these questions could be answered. Achievement in basic mathematical skills was measurable before and after enrollment in Mathematics 200. Since achievement in the course content was not dependent on basic skills, it was difficult to evaluate the effect of the remedial program on grades in the course. Attitude, on the other hand, became more favorable after exposure to mathematics content, as can be seen in Table V. A more positive attitude developed gradually as the remedial program, concurrent with the course progressed. Reactions of the students to the diagnostic tests varied from annoyance at still another bureaucratic "time-waster" to passive acceptance. Those who used the remedial materials and attended the clinic sessions tended to respond positively to remediation efforts at the college level. In fact, many expressed their gratitude after having concepts clarified.

Results to date from the investigation suggest several items of significance for educators:

1. Mathematics skills can be diagnosed and remediated on a small-group or individual basis.
2. It is at least equally important to try to modify negative attitudes toward mathematics.
3. From informal observation, not yet statistically evaluated, it appears that it might be wise to place students in an elementary classroom on several occasions prior to or concurrent with their involvement in the mathematics course. As a result of such experiences, the students appear to have greater understanding of the role of mathematics in elementary education and therefore greater motivation to become competent in this area.

Methods

In September 1970, all entering students to the College of Education were given the Wide-Range Achievement Test (Arithmetic only), the G-S-Z Diagnostic Test (Form A), the Suydam-Trueblood Attitude Toward Mathematics Scale, and a Self-Evaluation of Competence in Mathematics Scale (Appendix C-1, C-3, and C-4).

Tests were scored and the diagnostic profiles drawn. Students who scored below the mean on the G-S-Z Diagnostic Test (32 or below) were listed for remedial work. All data collected were subject to extensive statistical analysis, including item analysis, study of reliability, inter-test correlation, etc. Significant and relevant results of these analyses are included in the section on results. (The G-S-Z was machine-scored; all other tests were hand-scored. Results were then analyzed by computer.

With the Winter term subjects, a random sample was drawn from all of those with below-mean scores to make up an experimental group. The remaining "deficient" students made up the control group. This resulted in an experimental group of N=7, and a control group of N=8. Subjects in the experimental group were sent letters urging them to attend a weekly remedial clinic to improve their arithmetic skills (Appendix C-7). All students in the Mathematics 200 class were then retested at the end of the term with the same tests (G-S-Z Diagnostic, Suydam-Trueblood Attitude, and Self-Evaluation).

Since several of the Spring term students had not taken all of the tests, both sections of the Mathematics 200 course were completely retested at the beginning of the term. Inasmuch as too few of the experimental subjects sought remedial assistance in Winter term, the basic random selection method was retained, but a letter strongly implying that attendance at the clinic would be considered in the final course grade was sent to the experimental subjects (Appendix C-7). In the Spring term, the experimental group numbered 11 subjects, with N=9 in the control group. Another experimental design, suggested by a staff member at the CReWS office, was not followed for reasons given in Appendix B.

Use of the remedial texts provided on the part of the subjects was limited, although Mrs. Zemel used the material at clinic sessions. She also developed remedial worksheets in some of the skill areas for use in the clinic (Appendix C-6).

In summary, then, all Mathematics 200 students were given at least three tests before beginning the course, and were retested with the same tests at the end of the course. Experimental and control groups were selected by random sampling from those scoring below the mean on the Diagnostic test. Experimental subjects were urged mildly, and then more strongly, to attend a remedial clinic and to use the remedial texts available to them.

Results

The first task of the investigators was to develop and test a diagnostic arithmetic scale which could be machine-scored and which would provide information from which we could draw up a diagnostic profile (Appendix C-7).

Several forms of the diagnostic test were tried before Form A of the G-S-Z Diagnostic Test was given to 175 entering and current Education majors on September 24, 1970. Means, standard deviations, and reliability coefficients for the test, based on that sample, are given in Table I.

TABLE I

MEANS, STANDARD DEVIATIONS, AND RELIABILITY COEFFICIENTS FOR SUB-TESTS AND TOTAL TEST OF THE G-S-Z ARITHMETIC DIAGNOSTIC TEST, FORM A. (N=175)

Sub-test	Mean	Standard Deviation	Reliability*
1. Whole numbers	5.23	0.89	.194
2. Common fractions	4.71	1.20	.349
3. Decimals	5.13	1.04	.370
4. Percentages	3.35	1.46	.364
5. Relationships between fractions, decimals, and percentages	3.98	1.54	.526
6. Same as #5	2.83	1.75	.614
7. Rational-irrational numbers	1.77	1.40	.438
8. Exponents	1.76	1.64	.647
9. Simple algebra	3.04	2.45	.901
10. Simple geometry	0.79	1.16	.594
Total test	32.59	8.44	.804

*Kuder-Richardson 21 reliability (An error in the scoring key was later found to have reduced coefficients.)

The range of scores on this test, with a maximum of 60 possible points, was 12 to 55. The mean score was 32.59.

In the Fall term (1970), the diagnostic test was also administered to students enrolled in Mathematics courses other than Mathematics 200. Their

majors were in a variety of the colleges of the University. In Table II, the college or 2-year technology program is shown in addition to the number of students in each section, mean, standard deviation, and reliability coefficients. The purpose of this extended testing program was primarily to compare the performance on the test of prospective teachers and students in other types of programs.

TABLE II

MEANS, STANDARD DEVIATIONS, AND RELIABILITY COEFFICIENTS ON THE G-S-Z ARITHMETIC DIAGNOSTIC TEST, FORM A, FOR EDUCATION AND NON-EDUCATION STUDENTS

Course	Program	N	Means	Standard Deviations	Reliability Coefficients*
Math 801-1	2 yr. Tech.	21	32.24	6.64	0.537
Math 801-2	2 yr. Tech.	22	34.64	6.08	0.428
Math 801-3	2 yr. Tech.	22	29.45	5.88	0.432
Math 801-4	2 yr. Tech.	23	38.87	7.05	0.562
Math 20	Bus. Adm.	31	36.71	6.90	0.550
Math 10	Remed. Math	10	28.80	8.45	0.734
Math 63	Science	22	46.50	3.40	
Math 17	Lib. Arts	33	38.10	8.30	
	Education	175	32.59	8.44	0.804

*Kuder-Richardson 21 reliability formula.

As you can see, the mean score for Education majors was lower than for all but three of the eight comparison groups. The variability, as seen in the Standard Deviation column, was greater than for all but one of the comparison groups. These findings confirmed our unwritten hypothesis that Education majors, particularly those in Elementary Education, had a lower level of capability in arithmetic skills, but with great variation in skill, than students in other colleges of the University, despite the fact that they would eventually have to teach arithmetic to others.

A second question which had to be answered early in the investigation was the relationship of arithmetic skill, attitude toward mathematics, and

self-evaluation of competence in these skills. We had two measures of arithmetic skill since the Wide-Range Achievement Test, Arithmetic II, had also been given in September 1970.

TABLE III

COEFFICIENTS OF CORRELATION AMONG THE G-S-Z DIAGNOSTIC TEST, WRA ARITHMETIC TEST SUYDAM-TRUEBLOOD ATTITUDE TOWARD MATHEMATICS SCALE, AND SELF-EVALUATION OF COMPETENCE IN ARITHMETIC SKILLS (N=175)

	S-T Attitude	Self-Evaluation	G-S-Z
Suydam-Trueblood Scale			
Self-Evaluation	.604*		
WRA Arithmetic Test	.447*	.408*	.694*
G-S-Z Diagnostic Test	.436*	.446*	

*$p < .01$

Subsequent testing with other groups in the Elementary Education sequence confirmed the significant, if moderate, relationships among diagnostic, attitude, and self-evaluation scores. In an elementary education methods course (El.Ed. 326) Suydam-Trueblood Attitude Scale had a correlation coefficient of 0.583 ($p < .001$). In the Winter term Mathematics 200 course, N=33, relationships were again significant but moderate, as seen in Table IV. (Page 9.)

Although these coefficients are highly significant for Diagnostic and Attitude Pre-test, and for Diagnostic and Attitude Post-test, they do indicate a moderate relationship. Similarly, only a moderate relationship is found between the Attitude and Self-Evaluation scores, although one might expect a stronger one. The non-significant and low relationship between scores on the Diagnostic test and the Self-Evaluation (in pairs as Pre-tests or Post-tests) was not only a great surprise, but indicated a number of possibilities: (1) students' self-evaluation is unrealistic in

terms of their actual ability; (2) the Self-Evaluation scale is a poor test; (3) the two measures should be analyzed by some technique other than correlation.

TABLE IV

CORRELATION COEFFICIENTS OF PRE- AND POST-TEST SCORES ON THE G-S-Z DIAGNOSTIC, SUYDAM-TRUEBLOOD ATTITUDE, AND SELF-EVALUATION OF COMPETENCY SCALES (N=33).

	Diagnostic		Attitude		Comp.
	Pre-test	Post-test	Pre-test	Post-test	Pre-test
Diagnostic Post-test	0.6433**				
Attitude Pre-test	0.4872**	0.4710**			
Attitude Post-test	0.4890**	0.4712**	0.8565**		
Competency Pre-test	0.2116	0.3616*	0.5004**	0.4826**	
Competency Post-test	0.2900	0.3383	0.3555*	0.4289*	0.6985**

*p < .05
**p < .01

Inspection of the raw scores (Appendix A) reveals that the Self-Evaluation scores changed very little in the Post-test despite the substantial increases, in some cases, in actual competence as seen on the G-S-Z Diagnostic Post-test scores. In several cases (12, or one-third of the sample) as competence increased, self-evaluation of competence decreased. This enigmatic situation also occurred in connection with the attitude scores (for 8, or about one-fourth of the sample).

With the larger Spring '71 sample, N=74, further analysis of this situation was possible. Reliability of the G-S-Z Diagnostic test was .874 for this sample (as compared with .804 for the Winter '71 sample).

TABLE V

CORRELATION COEFFICIENTS OF PRE- AND POST-TEST SCORES
ON THE G-S-Z DIAGNOSTIC,
SUYDAM-TRUEBLOOD ATTITUDE, AND
SELF-EVALUATION OF COMPETENCE SCALES (N=96)

	Pre-test	Post-test	Pre-test	Post-test
Diagnostic Post-test	.661*			
Attitude Pre-test	.422*	.360*		
Attitude Post-test	.476*	.472*	.854*	
Self-Evaluation Pre-test	.347*	.380*	.602*	.559*

*$p < .01$

These scores are quite similar to those given in Table IV, and similarly suggest greater change in diagnostic scores than in attitude scores.

What were the actual changes in scores? Graphically, the shift in ranges and frequency distributions between the pre- and post-test scores on the diagnostic and attitude scales can be seen in Figures 1 and 2. (Page 11 and 12.) Changes in means can be seen in Table VI, on page 11.

Since the experimental groups in the Spring term sample also participated in the remedial activities in an inconsistent manner, despite the implied threat about their final grade, it is again problematic whether the increase in score observed can be attributed to remedial treatment (Table VII). The differences in means on the diagnostic test between experimental and control groups are insignificant.

Unfortunately, due to time schedules, there was no opportunity to administer the Self-Evaluation, Form A, a second time in the Spring term. The negative change in attitude scores, however, is not too surprising since these are students who were poor achievers in mathematics initially.

Our data suggest that there <u>is</u> improvement in the ability to handle mathematics problems, and overall in attitudes (that is, in the total sample), but that remedial treatment and/or exposure to the mathematics course had

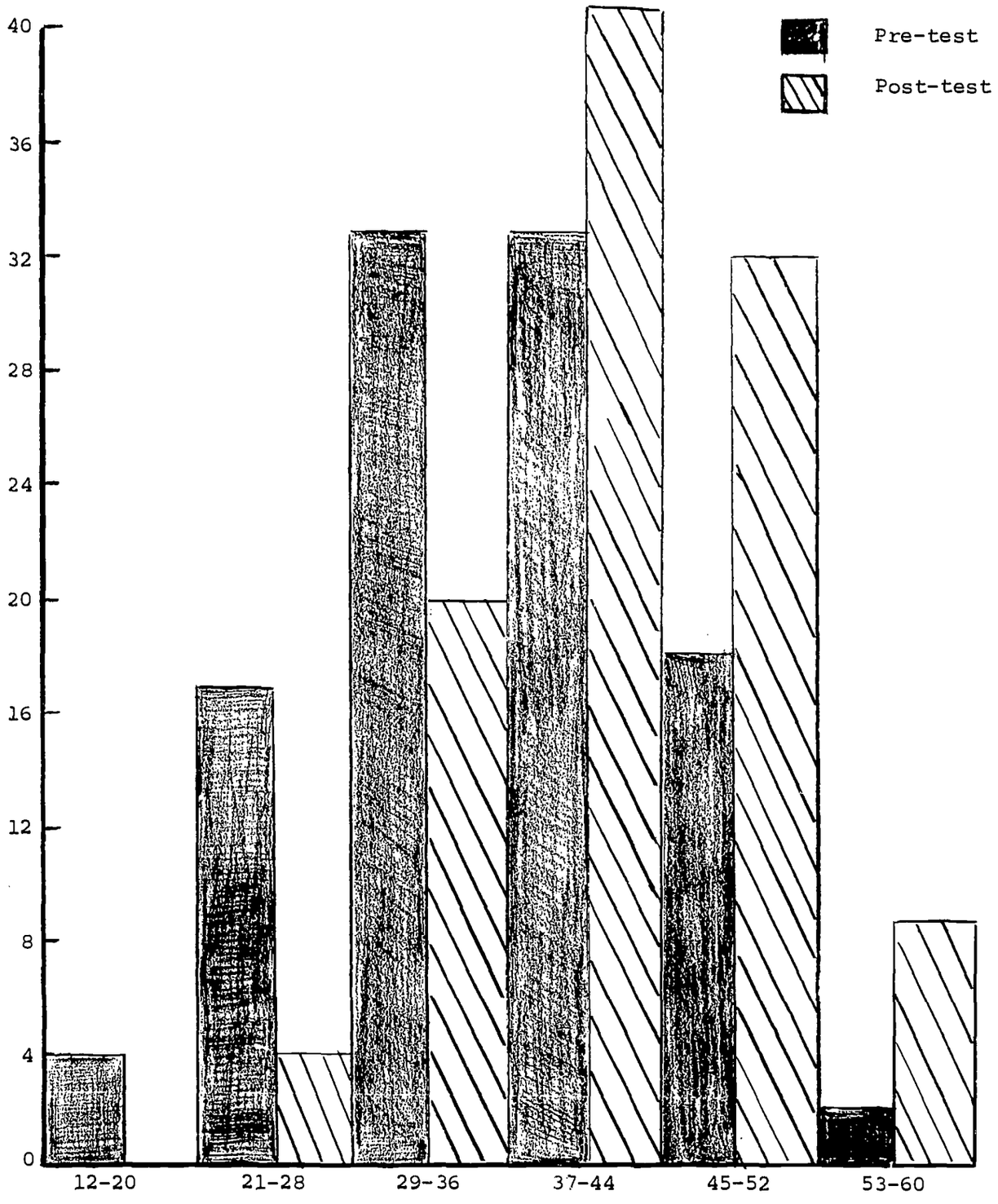


Figure 1
Frequency Distribution G-S-Z (A) N = 107

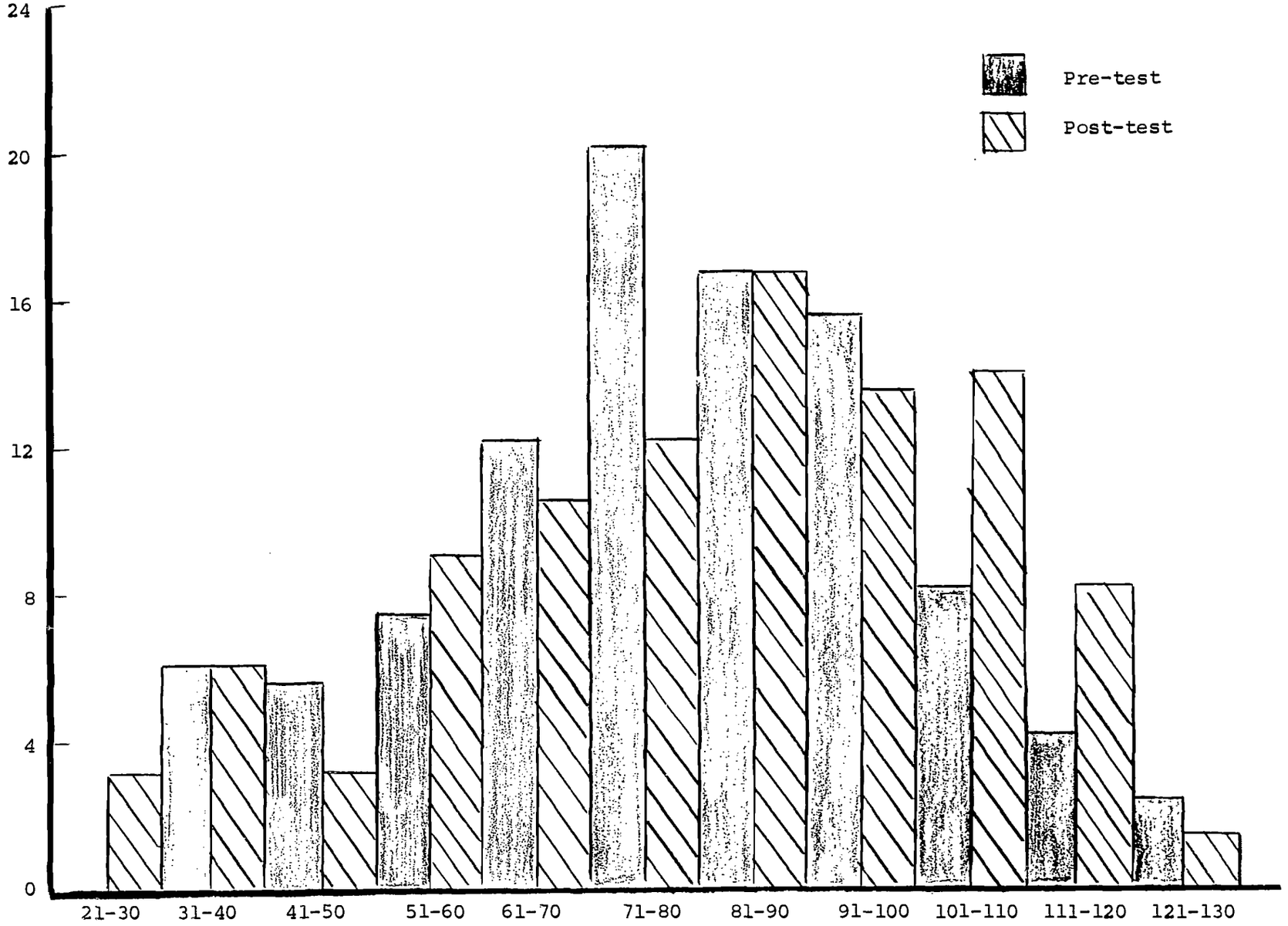


Figure 2
Frequency Distribution "Attitude Toward Mathematics" Scale (N = 107)

little effect on self-evaluation of competence. This may be due to an unrealistic self-perception at the outset, a lack of recognition of improvement of skills, or some combination of the two. It is apparently easier, however, to change surface expressions of attitude than to modify deeper-rooted self-concepts, even with demonstrated improvement and/or mastery.

TABLE VI

PRE-TEST AND POST-TEST MEANS ON THE G-S-Z DIAGNOSTIC, SUYDAM-TRUEBLOOD ATTITUDE, AND SELF-EVALUATION OF COMPETENCE SCALES FOR THREE GROUPS (WINTER '71, SPRING '71, AND TOTAL SAMPLE)

	Diagnostic			Attitude			Self-Evaluation		
	Pre	Post	t	Pre	Post	t	Pre	Post	t
Winter (33)	33.6	41.6	8.6*	74.8	81.1		24.0	24.5	
Spring (74)	37.2	42.5		76.7	77.9		23.2		
Total (107)	36.1	42.3		76.1	78.9		23.5		

*$p < .001$

TABLE VII

PRE- AND POST-TEST MEANS ON THE G-S-Z DIAGNOSTIC, SUYDAM-TRUEBLOOD ATTITUDE, AND SELF-EVALUATION OF COMPETENCE SCALES FOR EXPERIMENTAL AND CONTROL GROUPS (SPRING '71)

	N	Diagnostic		Attitude		Self-Evaluation	
		Pre	Post	Pre	Post	Pre	Post
Experimental	11	26.5	36.3	68.0	63.7	22.1(A)	15.4(B)
Control	7	28.6	38.3	61.3	60.0	20.4(A)	16.7(B)

A revision of the Diagnostic Scale, based on an item analysis of Form A, was administered to most of our sample in June 1971 (Appendix C-2). The new Form B has several notable modifications:

1. It is shorter (48 questions instead of 60);
2. It has fewer subtests (6 instead of 10);
3. It includes word problems, one in each of the subtests, which provide a seventh subtest measure which is listed separately as scale v;
4. It omits the simplest arithmetic operations, as well as geometry problems.

Reliability of the G-S-Z Diagnostic Scale, Form B, is estimated to be .820 (Kuder-Richardson 20).

The Self-evaluation scale was also revised, to match the subtests of Form B. The Winter term sample (N=33) was contacted by mail, with these students offered a nominal sum ($2.50) for their time to take the two revised tests. The Spring term sample was tested in the closing days of the term. For various personal reasons, not all members of the two populations responded. The results of administering the revised tests are shown in Table VIII, and the scores obtained with the revised tests are compared with the post-test Form A scores in Table IX.

TABLE VIII

MEANS OF REVISED DIAGNOSTIC AND SELF-EVALUATION SCALES FOR 2 SAMPLES, AND CORRELATION COEFFICIENTS FOR EACH SAMPLE

N	G-S-Z Form B	Self-Evaluation Form B	Correlation Coefficients
Winter (22)	29.45	18.55	.438
Spring (43)	28.26	18.21	.663

Table IX reflects the effects of the time lapse of three months between Form A post-test and Form B for the smaller sample in comparison with the lapse of only a few days between test administrations for the larger sample.

TABLE IX

CORRELATION COEFFICIENTS BETWEEN FORM A PRE-TEST, FORM A POST-TEST, AND FORM B POST-TEST SCORES FOR TWO SAMPLES

	Winter (22)	Spring (43)
G-S-Z (A) Pre-test x (A) Post-test	.635	.781
G-S-Z (A) Pre-test x (B) Post-test	.665	.833
G-S-Z (A) Post-test x (B) Post-test	.526	.786

It was believed, and the data bear this out, that the time lapse would be an important variable to consider. As anticipated, there was, for the smaller sample, a closer relationship between scores on the Form A pre-test and Form B than between the Form A post-test and Form B. This finding suggests that, for this sample, the time away from active involvement with mathematics erased many of the gains in competence made during the period when these students were taking the Mathematics 200 course. Even the addition of word problems in Form B (and the elimination of simple arithmetic and geometry in the revision) should not have created this situation, although scores for all subjects who took the Form B were lowest on subtest v (word problems) as compared to scores on other subtests. Wherever possible, these students will be retested in succeeding terms to see how they score after further time away from mathematics.

Attitude and self-evaluation scores were generally consistent, although marked individual variations were noted. Attitude scores tended to increase somewhat, suggesting some positive effect of an increased understanding of the concepts underlying arithmetic operations. There is some indication that students who have a higher Attitude score to begin with tend to make greater gains on the Self-Evaluation Test than those who start out with poorer Attitude scores.

To summarize, most of the data indicate statistically significant increases in scores from pre-test to post-test, but the expected strength of relationship among diagnostic, attitude, and self-evaluation of competence

scores was not as clear. Attempts to demonstrate the effectiveness of the remedial program failed because of the lack of participation of the experimental subjects in the program. The improvement of skill in these students can be attributed to the same forces which caused all other subjects in the study to improve: contact with mathematics and the personality of the instructor. To support this idea, we have statistical evidence which indicates that only about 50% of the variance can be accounted for by a regression equation which includes the G-S-Z pre-test score, the self-evaluation B score, and the difference in pre- and post-test Attitude scores, plus a constant. Other variables, such as those suggested, apparently account for the other 50% of variance. Attitudes toward mathematics improved overall to a degree, but not in proportion to the gain in skills. Self-evaluation appeared to be minimally affected by change in level of skill.

Conclusions

It is possible to design a diagnostic test for college students which will indicate strengths and weaknesses in fundamental arithmetic skills. Further, this can be done in a multiple-choice format, with or without the use of an IBM answer sheet. During the course of the project, the G-S-Z Diagnostic Test was revised again in an effort to increase its reliability (which varies with the reliability formula used as well as the particular "mix" of the subject group). Significant gains in basic skills do occur during a mathematics concepts course, but the durability of the improvement remains questionable.

Attitudes, too, improve slightly while taking the course, for most students. Some students, however, become more negative in their attitude toward mathematics. Although the test used is reliable, the question of validity always arises with attitude tests. We noted, for example, that some students always chose moderate agreement/disagreement with statements, while others had a remarkable number of "neutral" responses. Despite this difficulty, one can gain some idea of attitudes toward mathematics for the sample as a whole. Additionally, the multiple-choice format of this test lends itself to automatic scoring on an IBM answer sheet, and thus can be used in a mechanical sense with the diagnostic scale.

The self-evaluation of competence in arithmetic skills was linked in both Form A and Form B to the G-S-Z Diagnostic Test subtest titles. The results obtained with this test were puzzling. Students often felt, for example, that they could do decimal and percentage problems with complete confidence, yet made errors in more than half of the decimal and percentage problems. With a 1-to-4 self-rating scale, the self-evaluation can also be used with the IBM answer sheet, and in fact, with the development of the G-S-Z and self-evaluation Form Bs, all three tests could be answered on a single answer sheet, making computer analysis and even hand-scoring more feasible.

A difficulty found throughout the project with the experimental design was that there was no effective means of getting the experimental subjects to attend the remedial clinic which was established as part of the project. Since achievement in the mathematics course was not dependent on ability

to perform fundamental operations, and since the experimental subjects rarely attended remedial clinic sessions or used the recommended remedial materials, it was difficult to evaluate the effect of the remedial treatment on course grades or mathematics skills. Those students who did attend the clinic tended to react favorably, and their changes in attitude toward mathematics tended to be positive.

Investigation of mathematics programs currently in use in Pennsylvania elementary schools reveals no unique patterns of arithmetic competence which are incorporated into the Mathematics 200 course. Further study in coordinating needed competencies and course content is necessary.

We feel very strongly that competence, confidence, and positive attitudes are of crucial importance to the prospective teacher if he/she is to communicate effectively in the classroom. Despite the difficulties encountered in this investigation, the obvious need for a diagnostic-remedial program in this area mandates further study.

Recommendations

Our experiences during the investigation lead us to make several recommendations:

1. For lasting improvement in arithmetic skills, it may be necessary to drop the assumption that the students can perform fundamental operations at the outset of the number concepts course, and build explanations and practice of these skills into the course instead of omitting them. It appears that many of the "new math" programs fail to build the facility which comes with observing certain arithmetic patterns after doing many, many problems. This situation can be changed, i.e., increasing facility with fundamental operations, as part of the Mathematics 200 or a similar course.
2. Pre-college "new math" programs may contribute to more negative attitudes toward mathematics. This is seen in both the attitude and self-evaluation scores. This may also be a result of the lack of competence and/or confidence of the elementary school arithmetic teachers with whom our subjects had been in contact years earlier. To overcome the negative attitudes, more "success experiences" with mathematics are needed, as well as contact with instructors who have strong positive attitudes toward mathematics and communicate this as well as content.
3. The remediation portion of the program is indeed a problem with a commuting population and a varied class schedules. Since "threats" of lowered grades are ineffective, it is recommended that positive reinforcement in the nature of "points" toward the course grade or a small monetary fee be given to each student who attends each remedial clinic session and who completes remedial assignments.
4. It is possible that awareness of the role of arithmetic in the elementary classroom may modify attitudes toward mathematics. Classroom experience, therefore, should occur before or concurrently with the mathematics course. This may also help competence since the prospective elementary teacher will recognize the need to be able to perform fundamental operations quickly and correctly.

5. Some consideration should be given to extending the time period during which prospective teachers are involved with mathematics. A second pre-methods course might be required, or instruction might be required until the student reaches a prescribed level of mastery, in order to enter the methods course. It seems pointless to permit inadequately prepared students to enter a course in which they are to learn how to teach content which they themselves do not understand.

LIST OF REFERENCES

Buswell, G. T. A Comparison of Achievement in Arithmetic in England and Central California. Arith. Teach. 5: 1-9; Feb. 1958.

Callahan, Leroy G. A Study of Knowledge Possessed by Elementary School Teachers, In-Service and In-Training, of the Cultural, Psychological, and Mathematical Foundations of the Elementary School Mathematics Program. (Syracuse University, 1966) Dis. Abst. 27: 4149-50A; May-June 1967.

Combs, Louise. Summary of Study of Certification Requirements in Mathematics for Elementary Teachers in the 50 states. Am. Math Mon. 70: 428-433; Apr. 1963.

Committee on the Undergraduate Program in Mathematics. Preparation in Mathematics for Elementary School Teachers. Arith. Teach. 14: 198-199; Mar. 1967.

Creswell, John L. The Competence in Arithmetic of Perspective Georgia Elementary Teachers. Arith. Teach. 11: 248-250; Apr. 1964.

Creswell, John L. How Effective Are Modern Mathematics Workshops? Arith. Teach. 14: 205-208; Mar. 1967.

DeVault, M. Vere; Houston, W. Robert; and Boyd, C. Claude. Do Consultant Services Make a Difference? Sch. Sci. Math. 63: 285-290; Apr. 1963.

Dutton, Wilbur H. Attitudes of Prospective Teachers Toward Arithmetic. El. Sch. J. 52: 84-90; Oct. 1951.

Dutton, Wilbur H. Measuring Attitudes Toward Arithmetic. El. Sch. J. 55: 24-31; Sept. 1954.

Dutton, Wilbur H. Attitude Change of Prospective Elementary School Teachers Toward Arithmetic. Arith. Teach. 9: 418-424; Dec. 1962.

Dutton, Wilbur H. Individualizing Instruction in Elementary School Mathematics for Prospective Teachers. Arith. Teach. 13: 227-231; Mar. 1966.

Dutton, Wilbur H. and Cheney, Augustine P. Pre-Service and In-Service Education of Elementary School Teachers in Arithmetic. Arith. Teach. 11: 192-198; Mar. 1964.

Dutton, Wilbur H. and Hammond, H. Reginald. Two In-Service Mathematics Programs for Elementary School Teachers. Calif. J. Ed. R. 17: 63-67; Mar. 1966.

Fisher, John J. and others. Pre-Service and In-Service Education in Mathematics of Colorado Elementary School Teachers--A Status Report. Denver: Colorado State Department of Education, Oct. 1965.

Fulkerson, Elbert. How Well Do 158 Prospective Elementary Teachers Know Arithmetic? Arith. Teach. 7: 141-146; Mar. 1960.

Gee, Burton Cleon. Attitudes Toward Mathematics and Basic Mathematical Understanding of Prospective Elementary School Teachers at Brigham Young University. (Oregon State University, 1966.) Dis. Abst. 26: 6528; May 1966.

Griffin, John Duncan. North Carolina Elementary School Teachers' Understanding of Contemporary Arithmetic. (Duke University, 1966.) Dis. Abst. 27: 3616A; May-June 1967.

Hicks, Randall C. and Perrodin, Alex F. Topics in Mathematics for Elementary School Teachers. Sch. Sci. Math. 47: 735-744; Nov. 1967.

Kane, Robert B. Attitudes of Prospective Elementary School Teachers Toward Mathematics and Three Other Subject Areas. Arith. Teach. 15: 169-175; Feb. 1968.

Kenney, Russell A. Mathematical Understandings of Elementary School Teachers. Arith. Teach. 12: 431-442; Oct. 1965.

Kipps, Carol. Elementary Teachers' Ability to Understand Concepts Used in New Mathematics Curricula. Arith. Teach. 15: 367-370; Apr. 1968.

Nelson, L. Doyal and Worth, Walter H. Mathematical Competence of Prospective Elementary Teachers in Canada and in the United States. Arith. Teach. 8: 147-151; Apr. 1961.

Reys, Robert. Mathematical Competencies of Elementary Education Majors. J. Ed. Res. 61: 265-266; Feb. 1968a.

Reys, Robert E. Mathematical Competencies of Preservice Elementary School Teachers. Sch. Sci. Math. 68: 302-308; Apr. 1968b.

Reys, Robert and Delon, Floyd G. Attitudes of Prospective Elementary School Teachers Towards Arithmetic. Arith. Teach. 15: 364-366; Apr. 1968.

Shyrock, Jerry. A Mathematics Course for Prospective Elementary School Teachers. Arith. Teach. 10: 208-211; Apr. 1963.

Skypek, Dora Helen. A Comparison of the Mathematical Competencies of Education and Non-Education Majors Enrolled in a Liberal Arts College. Am. Math. Mon. 72: 770-773; Sept. 1965.

Smith, Lehi T. Curricula for Education of Teachers. Am. Math. Mon. 70: 202-203; Feb. 1963.

Smith, Frank. Prospective Teachers' Attitudes Toward Arithmetic. Arith. Teach. 11: 474-477; Nov. 1964.

Smith, Frank. How Well Are Colleges Preparing Teachers for Modern Mathematics?--An Answer. Arith. Teach. 14: 200-202; Mar. 1967.

Todd, Robert M. A Mathematics Course for Elementary Teachers: Does It Improve Understanding and Attitude? Arith. Teach. 13: 198-202; Mar. 1966.

Waggoner, Wilbur. Improving the Mathematical Competency of Teachers in Training. Arith. Teach. 5: 84-86; Mar. 1958.

APPENDIX A

RAW SCORES OF STUDENT SAMPLES

APPENDIX A-1

RAW SCORES ON G-S-Z DIAGNOSTIC TEST,
SUYDAM-TRUEBLOOD ATTITUDE SCALE, AND
SELF-EVALUATION OF COMPETENCE TESTS FOR WINTER '71 AND SPRING '71 SAMPLES

	G-S-Z Diagnostic			Attitude		Self-Evaluation		
Student	A-Pre	A-Post	B-Post	Pre	Post	A-Pre	A-Post	B-Post
101	25	32		36	30	23	24	
102	39	40		50	72	25	29	
103	42	39	33	97	98	22	27	18
104	43	42	31	96	106	23	22	18
105	25	46	30	76	92	21	22	16
106	28	44	36	89	93	26	27	19
107	28	43	22	64	36	28	28	19
108	44	47		77	81	23	21	
109	29	43	26	52	68	17	23	14
110	45	48	39	91	103	33	30	23
111	46	47	33	112	104	28	28	23
112	35	40	28	83	91	24	23	16
113	18	31	19	65	68	25	27	17
114	24	41		31	34	19	17	
115	23	36		34	55	21	26	
116	23	28	25	54	63	26	28	16
117	35	40		42	56	18	20	
118	30	25		52	69	24	22	
119	36	35		62	68	16	21	
120	39	50	28	113	118	28	25	21
121	19	29		80	72	22	16	
122	39	46		130	125	33	31	
123	33	42	26	72	88	25	25	16
124	45	48	39	103	103	27	32	22
125	30	51	34	97	113	30	29	22
126	33	50	18	71	84	31	27	22
127	37	48	32	95	102	29	21	17
128	37	44	26	73	28	19	18	14
129	27	38	18	92	87	24	23	18
130	28	36	31	103	101	18	20	17
131	48	59	41	71	102	25	32	25
132	34	39		95	107	22	27	
133	42	46	33	63	69	16	17	15
201	31	42	25	108	117	29		18
202	45	54	42	105	115	29		23
203	46	55	41	87	103	27		25
204	32	39	27	82	80	21		15
205	35	51	31	74	69	30		20
206	29	40	29	73	76	25		19

Student	G-S-Z Diagnostic A-Pre	A-Post	B-Post	Attitude Pre	Post	Self-Evaluation A-Pre	A-Post	B-Post
207	31	41	20	76	74	23		18
208	47	53	36	111	122	36		28
209	32	44	29	49	59	22		15
210	26	39	16	38	38	21		17
211	41	41	31	99	81	20		18
212	28	35	20	71	58	24		16
213	43	53	37	101	105	26		20
214	44	48	30	68	74	26		23
215	37	39	26	86	56	28		17
216	37	38	25	66	56	21		16
217	38	51	29	68	49	23		18
218	35	43	27	121	118	26		20
219	33	45	27	111	101	26		21
220	52	56	34	103	103	32		20
221	37	43	27	75	75	25		20
222	33	39	23	80	74	23		14
223	29	43	21	31	37	15		13
224	33	30	31	96	90	21		18
225	46	50	38	89	88	24		22
226	25	40	18	72	65	20		16
227	31	35	26	45	43	26		19
228	35	38	22	67	67	21		13
229	39	48	26	48	35	12		14
230	35	48	25	34	53	9		10
231	47	57	41	85	88	18		18
232	39	51	30	72	74	21		16
233	46	57	39	122	118	30		22
234	20	27	13	66	32	24		13
235	33	47	34	35	55	22		18
236	25	41	27	91	100	21		20
237	31	42	22	86	82	24		19
238	23	29	15	40	35	15		14
239	36	45	28	76	83	23		19
240	43	43	32	90	95	30		19
241	35	47	30	77	83	22		21
242	48	45	32	87	76	26		19
243	37	45	33	61	71	21		19
244	35	38		98	105	25		26
245	39	47		80	87	21		15
246	19	34		59	61	16		11
247	38	42		86	82	30		19
248	45	43		94	89	22		25
249	46	47		108	113	27		20
250	38	36		63	43	20		15
251	53	52		89	96	30		21
252	31	32		54	60	17		14
253	53	50		80	93	19		22

Student	G-S-Z Diagnostic A-Pre	A-Post	B-Post	Attitude Pre	Post	Self-Evaluation A-Pre	A-Post	B-Post
254	37	36		75	69	22		17
255	50	54		99	103	30		23
256	34	40		82	83	26		23
257	43	40		98	112	23		21
258	36	32		36	27	16		10
259	43	37		54	87	22		18
260	25	37		41	26	16		11
261	38	36		89	93	19		16
262	26	23		73	64	20		12
263	48	49		84	94	28		18
264	39	36		89	85	24		17
265	39	34		56	55	24		17
266	37	44		42	60	24		20
267	48	48		92	94	32		23
268	48	47		70	78	21		18
269	28	35		76	74	23		21
270	36	40		91	103	23		23
271	39	38		84	84	30		19
272	34	40		51	71	17		15
273	38	36		62	76	17		13
274	41	48		102	95	25		19

APPENDIX A-2

COMPARISON OF T SCORES ON POST-TESTS A AND B
OF THE G-S-Z DIAGNOSTIC TEST FOR ONE SAMPLE (N=43)

Student	Post-test A	Post-test B
1	51	57
2	56	53
3	43	47
4	41	45
5	60	51
6	48	48
7	46	48
8	67	58
9	48	48
10	43	43
11	30	54
12	58	64
13	44	35
14	37	47
15	41	41
16	56	47
17	56	45
18	68	69
19	60	53
20	65	66
21	26	28
22	54	58
23	46	48
24	47	41
25	29	31
26	51	50
27	48	56
28	54	53
29	51	56
30	48	38
31	47	45
32	64	70
33	65	69
34	43	47
35	60	54
36	44	51
37	44	38
38	63	61
39	50	50
40	43	31
41	54	54
42	37	38
43	61	63

APPENDIX B

DESCRIPTION OF REJECTED EXPERIMENTAL DESIGN

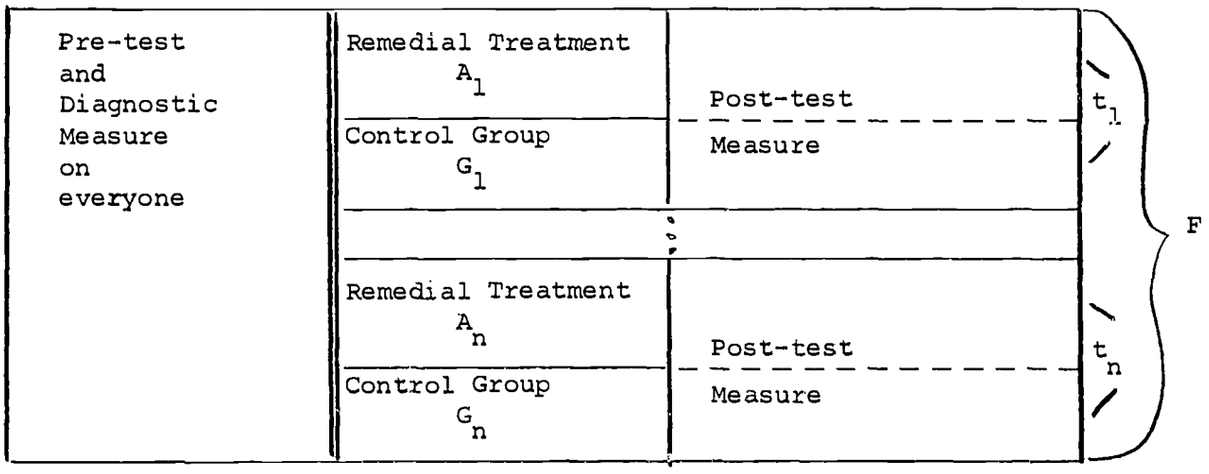
APPENDIX B

Although the investigators agreed with the basic model of an experimental design which follows, they felt that the fourth step, and the rationale therefore, which involved giving "neutral training" to the control group was something to which they could not agree. Giving unneeded remediation to the control group was ethically repugnant. It was believed that giving no remedial treatment to the control group would be preferable to offering remedial work from "OTHER treatment classifications." In the end, this is what was done. In effect, moreover, many of the experimental subjects, for reasons noted earlier in this report, were actually receiving no remedial treatment, but have still been considered as the experimental subjects because they were the only students notified of a special status.

MODEL OF AN EXPERIMENTAL DESIGN

John F. Howell
The Center for Cooperative
Research with Schools

Design No. 4



This experimental design is a relatively simple design yet allowing considerable control over unwanted sources of variation. The design is discussed in detail as Design No. 4 in "Experimental and Quasi-Experimental Designs for Research", Campbell and Stanley, in Gage's Handbook of Research on Teaching, Rand McNally, 1963.

The steps to follow are:

(1) Obtain a pre-test score on all subjects.
(2) Distribute the subjects to the various remedial treatment groups as their pre-test may indicate. The design above shows from one to "n" possible groups.
(3) Assign by RANDOM process half of the students in EACH remedial treatment group to the corresponding control group. For each treatment group there is a control group.
(4) Give remedial training to the treatment group and give neutral training to the control group.
(5) Obtain a COMMON post-test measure for the treatment and control groups. This measure should be the SAME as the pre-test measure.
(6) Calculate a DIFFERENCE SCORE; post-test minus pre-test.
(7) Compare the treatment group to the control group. This comparison can be a t-test if the number of treatment groups is small, say three or four. If the number of treatment groups exceeds five, then an analysis-of-variance

would be appropriate. In the latter case, some thought may be given to performing an analysis-of-covariance using the pre-test score as the covariate. A good reference for the three statistical analyses mentioned above would be Statistical Inference, Jerome Li, Edwards Bros., 1964, but there are many other references to those analyses.

The above design answers the question, "Does remedial treatment make a difference?" Since both the treatment and the control group have the same pre-test score (or the same diagnosis), any differences in the post-test measures can legitimately be attributed to the remedial treatment. A reasonable inference can be made that a correct diagnosis was made if remedial work corrected the difficulty. If this inference is to be truly tenable, then extreme care should be taken to insure that the various treatments be independent. This can be partially insured by providing the control groups with remedial work from OTHER treatment classifications. To do this with precision would require a much more complicated design and may well be considered in further development.

This stage of the study assumes that all psychometric considerations have been met.

One last caution; this design compares treatment-to-control groups and does not specifically insure that a correct diagnosis was made. It may well be true that any remedial treatment will improve test scores regardless of the diagnosis. To completely verify any diagnosis will require evidence that a person that is diagnosed for treatment A_1 will improve his test score for that and only that remedial treatment.

APPENDIX C

FACSIMILES OF MATERIALS

G-S-Z A R I T H M E T I C DIAGNOSTIC TEST
FORM A

Developed by A.Glaser, L.L.Schwartz, and J.Zemel
under OEG-2-700031(509), Sept.'70, PSU ABINGTON PA

DIRECTIONS: Solve each problem, using any available space on the page for scratchwork. Indicate the *one* correct answer in the appropriate space on the answer sheet. You will have 30 minutes in which to complete the test.

Last Name: _____

First Name: _____

Soc. Sec. No.: _____

Term Standing: 1 2 3 4 5 6 7 8 9 10 11 12
(Circle appropriate numeral)

Date: _____

(1) Add 46
37
+12

A. 94
B. 95
C. 105
D. 115
E. None of these

(2) Subtract . . . 204004
-96

A. 193 908
B. 203 908
C. 203 918
D. 204 008
E. None of these

(3) Divide . . .

$107\overline{)69978}$

A. 654
B. 663
C. 664
D. 666
E. None of these

(4) Divide . . .

$61\overline{)5917}$

A. 67
B. 77
C. 87
D. 96
E. None of these

(5) $2 \times 2548 + 5 = ?$

A. 5101
B. 6001
C. 6101
D. 6111
E. None of these

(6) Multiply 4093
× 109

A. 77 767
B. 445 137
C. 446 128
D. 446 137
E. None of these

(7) Add . . . $\frac{1}{2} + \frac{1}{3} = ?$

A. 1/5
B. 2/5
C. 4/6
D. 5/6
E. None of these

(8) Subtract . . . $8\frac{1}{4} - 7\frac{1}{3} = ?$

A. 29/12
B. 3/4
C. 11/12
D. 5/12
E. 10/12

(9) Subtract . . . $\frac{1}{7} - \frac{7}{50} = ?$

A. 43/350
B. 1/700
C. 3/100
D. 2/50
E. None of these

(10) Multiply . . . $3\frac{1}{3} \times 1\frac{1}{5} = ?$

A. $3\frac{1}{15}$
B. $3\frac{3}{5}$
C. $3\frac{4}{5}$
D. $4\frac{2}{3}$
E. None of these

(11) Divide . . . $\frac{1}{3} \div 3\frac{1}{6} = ?$

A. 2/19
B. $1\frac{1}{18}$
C. 5
D. $9\frac{1}{2}$
E. None of these

(12) What is the average of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{12}$ and $\frac{3}{4}$

A. 7/12
B. 5/9
C. 1/2
D. 2
E. None of these

(13) Subtract . . . 3.46 - 0.4 = ?

A. 2.06
B. 3.42
C. 3.16
D. 3.06
E. None of these

(14) Divide . . . 0.7 ÷ 0.1 = ?

A. 0.07
B. 0.7
C. 7
D. 70
E. None of these

(15) Multiply . . . 0.7 × 0.1 = ?

A. 0.07
B. 0.7
C. 7
D. 70
E. None of these

(16) Divide . . . 2.75 ÷ 2.5 = ?

A. 1/9
B. 0.11
C. $1\frac{1}{9}$
D. 1.1
E. None of these

(17) What is the average of .5, .7, and .6 ?

A. .06
B. .6
C. .9
D. 6
E. None of these

(18) Add . . . 9.097 + 0.003 = ?

A. 9.127
B. 9.137
C. 10.1
D. 10.0
E. None of these

(19) What is 5% of 360 ?

A. .018
B. 3.6
C. 7.2
D. 18
E. 72

(20) What is (1/3)% of 240 ?

A. 720
B. 80
C. 8
D. 0.8
E. None of these

(21) 24 is what percent of 72?

A. (1/3)%
B. 3%
C. 30%
D. $33\frac{1}{3}$%
E. None of these

(22) 2 is what percent of 400 ?

A. .005%
B. .05%
C. .5%
D. 5%
E. None of these

(23) 15 is 10% of what number ?

A. 1.5
B. 6.5
C. 15
D. 150
E. None of these

(24) 9 is 4.5% of what number ?

A. 20
B. 50
C. 100
D. 2000
E. None of these

(25) Which of A,B,C, and D is *not* equivalent to 5% ?

A. 1/20
B. .005
C. 5/100
D. 50/1000
E. each of these is equivalent to 5%

(26) Which of A,B,C, and D is *not* equivalent to $\frac{1}{2}$% ?

A. 1/200
B. 3/600
C. .005
D. 5/1000
E. each of these is equivalent to ½%

(27) Which of A,B,C, and D is *not* equivalent to $\frac{1}{3}$?

A. 3/10
B. $0.33333\overline{3}...$
C. $33\frac{1}{3}$ %
D. 4/12
E. each of these is equivalent to 1/3

(28) Which of A,B,C, and D is *not* equivalent to $\frac{1}{4}$?

A. 0.25
B. 25%
C. 3/12
D. 10/40
E. each of these is equivalent to 1/4

(29) Which of A, B, C, and D is *not* equivalent to 5.75 ?

A. 575%
B. $5\frac{3}{4}$
C. 575/100
D. 23/4
E. each of these is equivalent to 5.75

(30) Which of A, B, C, and D is *not* equivalent to 0.02 ?

A. 2/100
B. 1/50
C. 2%
D. 1/200
E. each of these is equivalent to 0.02

(31) What is the decimal equivalent of 3/40 ?

A. 0.0705
B. 0.075
C. 0.705
D. 0.75
E. None of these

(32) What is the decimal equivalent of 29.1 percent ?

A. 0.291
B. 2.91
C. 29.1
D. 291.0
E. None of these

(33) What common fraction is equivalent to 4.32 ?

A. 108/25
B. 104/25
C. 101/25
D. 12/5
E. None of these

(34) What common fraction is equivalent to 0.2 percent ?

A. 1/5
B. 1/50
C. 1/500
D. 1/5000
E. None of these

(35) What percent is equivalent to the number 5.5 ?

A. 550%
B. 220%
C. 55%
D. 5.5%
E. None of these

(36) What percent is equivalent to the decimal fraction 0.003 ?

A. (3/10)%
B. (3/100)%
C. (3/1000)%
D. (3/10000)%
E. None of these

(37) Which of $P=\sqrt{2}$ and $Q=1/17$ is (are) irrational ?

A. neither
B. P only
C. Q only
D. both

(38) Which of $P=\sqrt{2}$ and $Q=1/17$ has (have) a decimal equivalent that is infinitely long and non-repeating ?

A. neither
B. P only
C. Q only
D. both

(39) Which of $P=\pi$ and $Q=22/7$ is (are) rational ?

A. neither
B. P only
C. Q only
D. both

(40) Which of $P=-1$ and $Q=0$ is an integer ?

A. neither
B. P only
C. Q only
D. both

(41) Which of $P=0.3$ and $Q=0.3333\overline{3}...$ is (are) rational ?

A. neither
B. P only
C. Q only
D. both

(42) Which of $P=\sqrt{5}$ and $Q=\sqrt{9/4}$ is (are) rational ?

A. neither
B. P only
C. Q only
D. both

(43) 7^0 = ?

A. 0
B. 1
C. 7
D. 70
E. None of these

(44) $(0.1)^{-1}$ = ?

A. 0.01
B. 0.1
C. -0.1
D. 1
E. None of these

(45) $(\sqrt{5})(\sqrt{5})^{-1} = ?$

A. 1/5
B. -5
C. 0.5
D. 1
E. None of these

(46) $(0.1)^{-2} = ?$

A. -0.1
B. 0.001
C. 0.1
D. 2
E. None of these

(47) $\frac{2^{10}}{2} = ?$

A. 1
B. 10
C. 32
D. 512
E. None of these

(48) $(2^3)^3 = ?$

A. 5^3
B. 2^6
C. 2^9
D. 729
E. None of these

(49) Solve for n: $2+n = 8$

A. 6
B. -6
C. 4
D. -4
E. None of these

(50) Solve for n: $\frac{3n}{4} = 6$

A. 1
B. 2
C. 3
D. 4
E. None of these

(51) Solve for n: $4n-7 = 2n-3$

A. 1
B. 2
C. 3
D. 4
E. None of these

(52) Solve for n: $\frac{12}{n} = \frac{28}{7}$

A. 3
B. 4
C. 5
D. 7
E. None of these

(53) Solve for n: $\frac{24}{n} - 1 = 5$

A. 3
B. 4
C. 6
D. 8
E. None of these

(54) Solve for n: $6n-3 = -1$

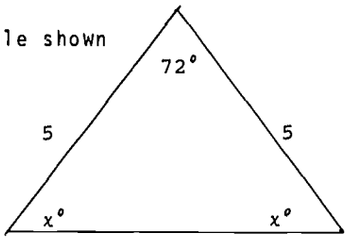
A. 2/3
B. 1/3
C. -1/3
D. -2/3
E. None of these

(55) Each angle of an equilateral triangle has a degree measure of ?

A. 45°
B. 50°
C. 60°
D. 75°
E. None of these

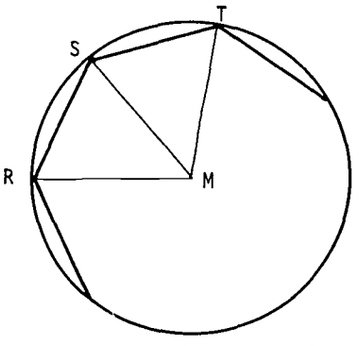
(56) For the triangle shown x = ?

A. 108
B. 54
C. 52
D. 49
E. None of these



The following information applies to each of the last four problems:

M is the center of the circle shown. A regular polygon of n sides is inscribed. The points R, S, and T are consecutive vertices of this polygon.



(57) If n=5, i.e., if the polygon is 5-sided, what is the degree measure of $\angle RMS$

A. 36°
B. 60°
C. 72°
D. 80°
E. None of these

(58) If n=5, what is the measure of $\angle RST$?

A. 75°
B. 90°
C. 108°
D. 120°
E. None of these

(59) If n=6, i.e., if the polygon is a hexagon, and if the radius of the circle is 10 inches long, how long is segment $\overline{ST}$?

A. 5 inches
B. 6 inches
C. 10 inches
D. 15 inches
E. None of these

(60) If $\angle RST$ measures 135°, then n = ?

A. 8
B. 7
C. 6
D. 4
E. None of these

G-S-Z A R I T H M E T I C DIAGNOSTIC TEST
FORM B

Developed by A. Glaser, L.L.Schwartz, and J.Zemel
under OEG-2-700031(509), May 1971 PSU ABINGTON PA

Last Name: _____

First Name: _____

Date: _____

DIRECTIONS: Solve each problem, using any available space on the page for scratchwork. Indicate the *one* correct answer in the appropriate space on the answer sheet. The 48 questions of this test are numbered from (61) to (108) inclusive.

(61) Add $\frac{2}{3} + \frac{2}{5} = ?$

A. 4/15
B. 1/2
C. 3/5
D. 16/15
E. None of these

(62) Add 9.097 + 0.03 = ?

A. 9.1
B. 9.127
C. 9.137
D. 10.0
E. None of these

(63) What is 5% of 360 ?

A. 1.8
B. 3.6
C. 7.2
D. 18
E. None of these

(64) What is the decimal equivalent of 3/40 ?

A. 0.0705
B. 0.075
C. 0.705
D. 0.75
E. None of these

(65) $7^0 = ?$

A. 0
B. 1
C. 7
D. 70
E. None of these

(66) Solve for n: $\frac{2n + 7}{2} = 5$

A. 2/3
B. 3/2
C. 2
D. 17/2
E. None of these

(67) One sixth of the girls at a certain campus have red hair. Two thirds of the students at that campus are girls. If the total number of students is 1800, how many of them are red-haired girls?

A. 100
B. 200
C. 300
D. 400
E. None of these

(68) Subtract 9.406 - 0.6 = ?

A. 3.406
B. 8.806
C. 9.34
D. 9.4
E. None of these

(69) What is (1/3)% of 240 ?

A. 720
B. 80
C. 8
D. 0.8
E. None of these

(70) What is the decimal equivalent of 29.1 percent ?

A. 0.291
B. 2.91
C. 29.1
D. 291.0
E. None of these

(71) $(0.1)^{-1} = ?$

A. 0.01
B. 0.1
C. -0.1
D. 1
E. None of these

(72) Solve for n: $\frac{3n}{10} + 7 = 1$

A. -23
B. -20
C. 1
D. 80/3
E. None of these

(73) Subtract $\frac{1}{7} - \frac{7}{50} = ?$

A. 1/700
B. 1/350
C. 3/70
D. 43/350
E. None of these

(74) What is six tenths of two tenths ?

A. 0.012
B. 0.12
C. 1.2
D. 0.03
E. None of these

(75) 24 is what percent of 72 ?

A. (1/3) percent
B. 3 percent
C. 30 percent
D. $33\frac{1}{3}$ percent
E. None of these

(76) What common fraction is equivalent to 4.32 ?

A. 108/25
B. 104/25
C. 101/25
D. 12/5
E. None of these

(77) $(\sqrt{5})(\sqrt{5})^{-1} = ?$

A. 1/5
B. -5
C. 0.5
D. 1
E. None of these

(78) Solve for n: $7 - 6n = 3(n+5)$

A. -22/9
B. -8/9
C. 2/9
D. 3/8
E. None of these

(79) Subtract $8\frac{1}{4} - 7\frac{1}{3} = ?$

A. 7/12
B. 3/4
C. 5/6
D. 11/12
E. None of these

(80) Divide $0.7 \div 0.1 = ?$

A. 0.07
B. 0.7
C. 7
D. 70
E. None of these

(81) Mary paid $18.00 for a dress that had been reduced in price by 10%. What would the cost have been without this reduction?

A. $18.18
B. $19.80
C. $20.00
D. $36.00
E. None of these

(82) What common fraction is equivalent to 0.2 percent ?

A. 1/5
B. 1/50
C. 1/500
D. 1/5000
E. None of these

(83) $(0.1)^{-2} = ?$

A. -0.1
B. 0.001
C. 0.1
D. 100
E. None of these

(84) Solve for n: $\frac{12}{n} = \frac{28}{7}$

A. 3
B. 4
C. 5
D. 7
E. None of these

(85) Multiply $3\frac{1}{3} \times 1\frac{1}{5}$ = ?

A. 14/15
B. 11/15
C. 20/9
D. 4
E. None of these

(86) Multiply 0.7 × 0.2 = ?

A. 0.014
B. 0.14
C. 1.4
D. 14
E. None of these

(87) 2 is what percent of 400?

A. 0.005 percent
B. 0.05 percent
C. 0.5 percent
D. 5 percent
E. None of these

(88) Given that: S = 2/3, T = 0.666, and U = 67/100. In what order will these three numbers appear, if they are arranged from least to greatest?

A. TUS
B. TSU
C. SUT
D. STU
E. None of these

(89) $\frac{2^{10}}{2}$ = ?

A. 1
B. 10
C. 32
D. 512
E. None of these

(90) Solve for n: $\frac{24}{n} - 1 = 5$

A. 3
B. 4
C. 6
D. 8
E. None of these

(91) Divide $\frac{1}{3} \div 3\frac{1}{6}$ = ?

A. 2/19
B. 19/18
C. 2/3
D. 19/2
E. None of these

(92) Divide 2.75 ÷ 2.5 = ?

A. 1/9
B. 0.11
C. 10/9
D. 1.1
E. None of these

(93) 15 is 30% of what number ?

A. 45
B. 50
C. 200
D. 450
E. None of these

(94) What percent is equivalent to the number 5.5 ?

A. 550 percent
B. 220 percent
C. 55 percent
D. 5.5 percent
E. None of these

(95) How many seconds are there in 3600 hours ?

A. 60^2
B. 60^3
C. 60^4
D. 60^5
E. None of these

(96) Solve for n: $\frac{1}{3n - 2} + 7 = \frac{1}{3}$

A. -15/22
B. -1/20
C. 37/60
D. 41/66
E. None of these

(97) What is the average of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{12}$, and $\frac{3}{4}$?

A. 7/12
B. 3/4
C. 1/2
D. 2
E. None of these

(98) What is the average of .5, .7, and .6 ?

A. 0.06
B. 0.6
C. 0.9
D. 6.0
E. None of these

(99) 9 is 4.5 percent of what number ?

A. 20
B. 50
C. 100
D. 2000
E. None of these

(100) What percent is equivalent to the decimal fraction 0.003 ?

A. (3/10) percent
B. (3/100) percent
C. (3/1000) percent
D. (3/10000) percent
E. None of these

(101) $(2^3)^3$ = ?

A. 5^3
B. 2^6
C. 2^9
D. 729
E. None of these

(102) How old is Ann now, if she is one third as old as her mother, but will be one half as old as her mother in 14 years?

A. 14
B. 15
C. 16
D. 17
E. None of these

(103) $\frac{6}{5} \times (\frac{1}{2} + \frac{2}{3})$ = ?

A. 2/5
B. 12/25
C. 36/35
D. 7/5
E. None of these

(104) $(0.2 \times 1.7) + (0.2 \times 0.3)$ = ?

A. 0.3
B. 0.4
C. 0.94
D. 2.4
E. None of these

(105) What is 12½% of 8% of 200 ?

A. 2
B. 4.1
C. 20
D. 41
E. None of these

(106) Which of A,B,C, and D is *not* equivalent to (1/2)% ?

A. 1/200
B. 3/600
C. 0.005
D. 5/1000
E. each of these is equivalent to ½%.

(107) $(2^4+2^3+2^2+2^1+2^0)$ = ?

A. 2^9
B. 2^{10}
C. 30
D. 31
E. None of these

(108) $(a-b)^2$ = ?

A. $a^2 - b^2$
B. a^2-ab+b^2
C. a^2-ab-b^2
D. $a^2-2ab+b^2$
E. None of these

Name: _____

ATTITUDE TOWARD MATHEMATICS

Date: _____

Marilyn N. Suydam and Cecil R. Trueblood

This is to find out how you feel about mathematics. You are to read each statement carefully and decide how *you* feel about it. Then indicate your feeling by marking on the line before each question:

A - if you strongly agree
B - if you agree
C - if your feeling is neutral
D - if you disagree
E - if you strongly disagree

_____1. Mathematics often makes me feel irritable and angry.

_____2. I usually feel happy when doing mathematics problems.

_____3. I think my mind works well when doing mathematics problems.

_____4. When I can't figure out a verbal problem, I feel as though I am lost in a mass of words and numbers and can't find my way out.

_____5. I avoid mathematics because I am not very good with numbers.

_____6. Mathematics is a stimulating and interesting subject.

_____7. My mind goes blank and I am unable to think clearly when working mathematics problems.

_____8. I feel sure of myself when doing mathematics.

_____9. I sometimes feel like running away from my mathematics problems.

_____10. When I hear the word mathematics, I have a feeling of dislike.

_____11. I am afraid of mathematics.

_____12. Mathematics is fun.

_____13. I like anything with numbers in it.

_____14. Mathematics problems often scare me.

_____15. I usually feel calm when doing mathematics problems.

_____16. I feel good toward mathematics.

_____17. Mathematics tests always seem difficult.

_____18. I think about mathematics problems outside of class and like to work them out.

_____19. Trying to work mathematics problems makes me nervous.

_____20. I have always liked mathematics.

_____21. I would rather do anything else than do mathematics.

_____22. Mathematics is easy for me.

_____23. I dread mathematics.

_____24. I feel especially capable when doing mathematics problems.

_____25. Mathematics class stimulates me to look for ways of applying mathematics to solving practical problems.

_____26. Time drags in a mathematics lesson.

AG|21|00|70|

SELF-EVALUATION OF COMPETENCE IN MATHEMATICS

for project OEG-2-700031(509)
AG, LLS, & JZ, Sept. '70 PSU

Last Name: _____

First Name: _____

Date: _____

DIRECTIONS: Nine areas of mathematics are listed below. For each please indicate (using a check or X) how confident you feel that you have mastered that area of mathematics. Be honest rather than wishful!

AREAS OF MATHEMATICS	NO CONFIDENCE	LITTLE CONFIDENCE	MUCH CONFIDENCE	COMPLETE CONFIDENCE
(a) WHOLE NUMBERS: +, −, ×, and ÷				
(b) COMMON FRACTIONS: +, −, ×, and ÷				
(c) DECIMALS: +, −, ×, and ÷				
(d) PERCENTAGES				
(e) RELATIONSHIP BETWEEN COMMON FRACTIONS, DECIMALS, and PERCENTAGES				
(f) CLASSIFICATION OF NUMBERS INTO RATIONAL AND IRRATIONAL NUMBERS				
(g) EXPONENTS				
(h) SIMPLE ALGEBRA: Solving first degree equations in one unknown				
(i) SIMPLE GEOMETRY: Polygons and their angles				

AG |20|09|1970|

3

[illegible]

[illegible] MATHEMATICS

[illegible] May 19[illegible]

[illegible]

[illegible] Seven areas of mathematics [illegible] for each, please indicate [illegible] the [illegible] how [illegible] you feel that you have mastered that area of mathematics. Be as honest, rather than [illegible] as [illegible]

[illegible] confidence

[illegible]

[illegible] confident

[illegible] confidence

Answer spaces (31) through (3[illegible]) have been reserved [illegible] [illegible]ATION.

Answer Space Number	Area	
(31)	(q)	Common fractions: [illegible]
(3[illegible])	(r)	[illegible]: [illegible]
(33)	(s)	PERCENTAGES
(34)	(s)	[illegible] COMMON FRACTIONS, DECIMALS, and PERCENTAGES
(35)	(t)	EXPONENTS
(36)	(u)	SIMPLE ALGEBRA: Solving first degree equations in one unknown.
(3[illegible])	(v)	[illegible] PROBLEMS

ADDING AND SUBTRACTING FRACTIONS

It is very easy to add or subtract fractions when the denominators are the same. For example, $\frac{1}{7}+\frac{3}{7}=\frac{4}{7}$. When the denominators are different, we find a common denominator and change each fraction to a new one with that denominator. For example $\frac{1}{4}+\frac{3}{5}=\frac{5}{20}+\frac{12}{20}=\frac{17}{20}$. In the latter example 20 is divisible by both 2 and 5. We can always find a common denominator by simply multiplying the denominators in the problem. However, it is desirable to use numbers as small as possible for the denominators (1) to keep the arithmetic easy and (2) to avoid a longer reducing process at the end. For example: $\frac{1}{9}+\frac{2}{15}+\frac{15}{45}+\frac{13}{135}=\frac{15}{135}+\frac{18}{135}+\frac{65}{135}=\frac{98}{135}$ or $\frac{14}{45}\times\frac{3}{3}+\frac{11}{45}=\frac{42}{135}+\frac{55}{135}=\frac{97}{135}$

(note: $9\times 15=135$)

In contrast:

$$\frac{1}{9}+\frac{2}{15}=\frac{5}{45}+\frac{6}{45}=\frac{11}{45}$$

Therefore, we look for the least common denominator (L.C.D.) i.e. the smallest number which is divisible by all of the denominators in the problem. Sometimes the L.C.D. can be discovered by inspection, but sometimes it is not so obvious. Thus a method shall be indicated to find the L.C.D.

Note: $\frac{a}{b}=\frac{a\times c}{b\times c}$ when $c=0$, for example $\frac{4}{7}=\frac{4\times 3}{7\times 3}=\frac{12}{21}$

Example 1.

$$\frac{5}{18}+\frac{1}{30}+\frac{3}{40}=\frac{5}{2\times 3\times 3}+\frac{1}{2\times 3\times 5}+\frac{3}{2\times 2\times 2\times 5}$$

The L.C.D. in this problem is $2\times 2\times 2\times 3\times 3\times 5=360$. It is obtained by taking a representative of each factor and writing it the maximum number of times it appears in any denominator. The denominator 18 has 2 as a common factor once. The denominator 30 also has 2 as a factor once. However, the denominator 40 has the factor 2 appearing 3 times. Therefore, $2\times 2\times 2$ is in the L.C.D.

$$\frac{5\times 2\times 2\times 5}{2\times 3\times 3\times 2\times 2\times 5}+\frac{1\times 3\times 2\times 2\times 3}{2\times 3\times 3\times 2\times 2\times 5}+\frac{3\times 3\times 3}{2\times 3\times 3\times 2\times 2\times 2\times 5}=\frac{100}{2\times 2\times 2\times 3\times 3\times 5}+\frac{12}{2\times 2\times 2\times 3\times 3\times 5}+\frac{27}{2\times 2\times 2\times 3\times 3\times 5}$$

$$\frac{139}{2\times 2\times 2\times 3\times 3\times 5}=\frac{139}{360}$$

In order to reduce the fraction $\frac{139}{360}$, we would have to cancel common factors. The only common factors to both the numerator and denominator would be 2, 3, 5. But since 139 is not divisible by either 2, 3, or 5, the fraction cannot be reduced.

Example 2: $\frac{1}{16}+\frac{1}{30}+\frac{1}{75}=\frac{1}{2\times 2\times 2\times 2}+\frac{1}{2\times 3\times 5}+\frac{1}{3\times 5\times 5}=\frac{1\times 3\times 5\times 5}{2\times 2\times 2\times 2\times 3\times 5\times 5}+\frac{1\times 2\times 2\times 2\times 5}{2\times 2\times 2\times 2\times 3\times 5\times 5}+\frac{1\times 2\times 2\times 2\times 2}{2\times 2\times 2\times 2\times 3\times 5\times 5}=$

$$\frac{25+15+6}{2\times 2\times 2\times 2\times 3\times 5\times 5}=\frac{46}{2\times 2\times 2\times 3\times 5\times 5}=\frac{23}{1\times 2\times 2\times 3\times 5\times 5}=\frac{23}{300}$$

Do the following problems:

1) $\frac{3}{20}+\frac{1}{25}-\frac{7}{50}$ 2) $\frac{2}{45}+\frac{4}{21}-\frac{1}{175}$ 3) $\frac{5}{72}-\frac{1}{84}-\frac{3}{16}$

MULTIPLYING FRACTIONS

To multiply two fractions, one must multiply the numerators and multiply the denominators.

For example: $\frac{1}{7} \times \frac{2}{3} = \frac{1 \times 2}{7 \times 3} = \frac{2}{21}$

Sometimes, the answer can be reduced. For example:

$\frac{1}{2} \times \frac{2}{7} = \frac{1 \times \cancel{2}^{1}}{\underset{1}{\cancel{2}} \times 7} = \frac{1}{7}$

The process commonly called "cancellation" is simply the "removal" of common factors of the numerator and denominator prior to carrying out the multiplication.

$\frac{1}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{7} = \frac{1 \times 1}{1 \times 7} = \frac{1}{7}$

"Removal of a common factor" of both numerator and denominator means to divide both by that factor. In the above example, when 2 is divided by 2, the answer is 1.

Note that the following example is incorrect. Why? *

$\frac{2 + 3}{2 \times 5} = \frac{4}{5}$

Do the following problems:

1.) $\frac{3}{4} \times \frac{8}{21} =$

2.) $\frac{1}{7} \times \frac{21}{25} \times \frac{1}{3} =$

3.) $\frac{5}{48} \times \frac{3}{15} \times \frac{8}{21} =$

* To reduce a fraction means to "cancel" common factors. Although 2 is a factor of the denominator, it is not a factor of the numerator and therefore cannot be cancelled.

The Pennsylvania State University
1600 Woodland Road
Abington, Pa. 19001
Dec. 21, 1970

Dear

Enclosed you will find a profile sheet giving your scores on the math diagnostic test you took earlier this year. A total score below 32 and/or subtest scores below 4 indicate the need for remedial work on your part.

Three texts are listed on the profile sheet which are on reserve in the Ogontz Campus Library. Although they each provide remedial help in the various mathematical skills, each author uses a slightly different approach. Use the one which is most helpful to you. We suggest that you begin your efforts, if needed, before taking any math courses. In addition, Mrs. Zemel and Dr. Glaser will be available for assistance if the suggested texts leave you still confused.

Sincerely yours,

Lita L. Schwartz
Anton Glaser
Jacqueline Zemel

Project No. OB058
Grant No. OEG-2-7000-31 (509)

THE PENNSYLVANIA STATE UNIVERSITY
THE OGONTZ CAMPUS
1600 Woodland Road
Abington, Pennsylvania 19001

Area Code 215
TU 6-9400

April 5, 1971

Dear

On the mathematics test which you took in the College of Education, your scores indicate weaknesses in mathematical skills. A copy of your score profile is enclosed. Since, as a teacher, you will have to teach these skills to your students, we consider it most important that your level of competence be increased.

A remedial clinic will be conducted by Mrs. Zemel on Mondays at Common Break, in Room 308. Your participation in the clinic will be considered in the determination of your final grade.

In addition, there are several copies each of three remedial books on reserve in the campus library (Rm. 119). The titles are listed on your profile sheet. The Heddens and Heywood texts have very clear Tables of Contents, enabling you to find material in your areas of weakness quite easily. In the Minnick-Strauss text, the topics are not organized in the same way. Therefore, if you wish to use that book, Mrs. Zemel will work out specific workpage assignments with you in the clinic. Each of these texts has a different approach and each has several advantages for the learner.

The first clinic session will be held on Monday, April 12th.

Sincerely yours,

Lita L. Schwartz, Project Director
Anton Glaser
Jacqueline Zemel

Math 200 Project
OEG-2-700031 (509)

1600 Woodland Road
Abington, Penna.
May 31, 1971

Dear

As you know, we have been working diligently on a project to elevate the arithmetic skills of prospective elementary school teachers. As part of your Math 200 course in Winter term, you took a number of tests for the project. Whether or not you are an Elementary Education major, we would appreciate your help in another aspect of the project.

We would appreciate it if you would come to Room 313 for about 30 minutes to take a revised form of one of the tests. Preferred times are:

Tuesday, June 8	12:45 - 1:15
Tuesday, June 15	12:45 - 1:15

If these are both inconvenient, please stop in or call for an appointment at another time. For reinforcement, you will be paid $2.50 for your efforts. A sign-up sheet for the dates given will be posted outside Room 313.

Thank you very much.

Sincerely yours,

Lita L. Schwartz

Lita L. Schwartz
Project Director

LLS:w